Assuming $h_1 < h$, let us consider the rectangle $|x| \leq h_1$, $|y| \leq 1$. When $|x| < h$, we can differentiate (5.1) any number of times with respect to both variables, hence

$$u|_{|x|=h_1} = 0, \qquad u_{xx}|_{|x|=h_1} = 0$$

is true.

From this we infer, using the generalized condition of orthogonality of (1.9) from [5], that $c_k \cos \lambda_k h_1 = 0$, i.e. $c_k = 0$ $(k = 1, 2, \ldots)$. So that a nontrivial expansion of a zero is impossible, and this completes the proot ot uniqueness.

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ROLLING OF ELASTIC BODIES

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A study is made of the rolling of an elastic cylinder on an elastic foundation. The deformation of the bodies precludes the pure rolling of one body on the other. The rolling is accompanied by sliding. Some recent investigations contain results concerning the rolling of bodies with identical elastic properties. The earliest investigations in this area were conducted by Petrov [1] and Reynolds [2]. This problem was later studied by Fromm [3], who confines himself to the application of Hertz's results [4]. The resistance to rolling of a rigid body on an elastic and inelastic foundation was also investigated by Ishlinskii [5]. The papers of Glagolev [6] and Desoyer [7] contain the general equations for the investigation of the rolling resistance of elastic bodies with different elastic constants. Glagolev solved this problem for bodies with identical elastic constants and examined the limiting case. Desoyer obtained a singular integral equation for the general case and examined this limiting case.

1. Herein, no restrictions are imposed on the elastic properties of the cylinder or the

foundation. It is assumed that the contact region consists of a slipping region and a sticking region. The rolling process is assumed to be quasi-static and elastic.

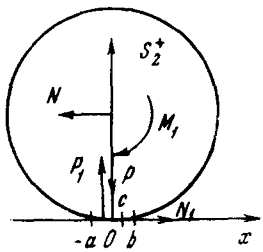

Fig. 1

Assume that the contact line is small in comparison with the body measurements and that surface irregularities are sufficiently small so that the bodies may be considered smooth. The scheme under study is shown in Fig. 1. The cylinder is acted upon by a moment M, a vertical force P and a horizontal force N. The force N is equal and opposite in direction to the traction at the foundation surface N_1, in the direction of motion. Also acting on the cylinder is the normal reaction of the foundation $P_1 = P$. The ends of the line of contact will be denoted by a and b. Since the length of the contact line is small in comparison with the dimentions of the bodies, the boundary conditions may be taken along the straight line $y = 0$ and the two bodies may be replaced by two half-planes. All quantities pertaining to the lower half-plane will be designated with the subscript 1 while those pertaining to the upper half-plane will be designated by the subscript 2.

Suppose that the contact line of the lower half-plane is acted upon by a normal load $Y^-_{y1}(x) = -P(x)$ and a tangential load $X^-_{y1}(x) = T(x)$. Correspondingly, the upper half-plane is acted upon by $Y^+_{y2}(x) = -P(x)$ and $X^+_{y2}(x) = T(x)$. We assume that the loads and torques vanish at infinity. The boundary conditions for our problem then are:

1) the normal and tangential loads are equal along the entire contact line

$$Y_{y1}^-(x) = Y_{y2}^+(x), \qquad X_{y1}^-(x) = X_{y2}^+(x) \tag{1.1}$$

2) the normal displacements along the contact line are interrelated by

$$v_1^-(x) - v_2^+(x) = {}^1/_2\, x^2 / R \tag{1.2}$$

3) the contact line is broken down into a slipping region $(-a, c)$ and a sticking region (c, b). In these regions, the boundary conditions are

$$T(x) = \nu P(x), \qquad u_1^-(x) = u_2^+(x) \tag{1.3}$$

where ν is the coefficient of Coulomb friction.

Let us formulate the problem with the above stated boundary conditions.

Let $\Phi_1(z)$ be a piecewise analytic function defined in the lower half-plane. This function may be continued along the unloaded region of the x-axis to the upper half-plane. Similarly, we define the function $\Phi_2(z)$ in the upper half-plane. It is known that the stresses and displacements are related to the functions $\Phi_1(z)$ and $\Phi_2(z)$ by the following relations [8]:

$$Y_{yk}(z) - iX_{yk}(z) = \Phi_k(z) - \Phi_k(\bar{z}) + (z - \bar{z})\overline{\Phi_k'(z)} \tag{1.4}$$

$$(k = 1, 2)$$

$$2\mu_k(u_k'(z) + iv_k'(z)) = \chi_k\Phi_k(z) + \Phi_k(\bar{z}) - (z - \bar{z})\overline{\Phi_k'(z)} \tag{1.5}$$

Satisfying (1.1), we can prove that

$$\Phi_1(z) = -\Phi_2(z) \tag{1.6}$$

From (1.2) and (1.3), we obtain the relation for determining $\Phi_1(z)$. In terms of $\Phi_1(z)$, (1.2) takes the form

$$k_2\Phi_1^+(x) - k_1\overline{\Phi}_1^+(x) + k_2\Phi_1^-(x) - k_1\overline{\Phi}_1^-(x) = ix / R \qquad (1.7)$$

$$\left(k_1 = \frac{\chi_1}{4\mu_1} + \frac{1}{4\mu_2}, \quad k_2 = \frac{\chi_2}{4\mu_2} + \frac{1}{4\mu_1}\right)$$

This condition holds over the entire contact length. In each of the regions of sticking and slipping there is an additional condition for the determination of $\Phi_1(z)$. These conditions take the form

$$k_2\,\Phi_1^+(x) + k_1\overline{\Phi}_1^+(x) + k_2\,\overline{\Phi}_1^-(x) + k_1\Phi_1^-(x) = 0 \qquad (1.8)$$

$$(\nu + i)\,\Phi_1^+(x) - (\nu - i)\,\overline{\Phi}_1^+(x) = (\nu + i)\,\Phi_1^-(x) - (\nu - i)\,\overline{\Phi}_1^-(x) \qquad (1.9)$$

Thus, over the entire contact region there are two conditions for each of functions $\Phi_1(z)$ and $\overline{\Phi}_1(z)$ to be determined. We have thus obtained the relations with piecewise constant coefficients for determining $\Phi_1(z)$ and $\overline{\Phi}_1(z)$.

2. It is known that this homogeneous problem with piecewise constant coefficients is equivalent to a problem in the theory of linear differential equations. For the given conditions on the contact boundary, we can construct a differential equation of the Fuchsian type with three singular points. Let us transform the complex z plane into the complex w plane in such a way that the points a, c, b will correspond to $0, 1$ and ∞, respectively, in the w plane. Gauss' hypergeometric equation takes the form [9]

$$w(1-w)\;\Phi_1''(w) + (1 + \varphi/\pi - 2w)\,\Phi_1'(w) - ({}^1/_4 + \beta_1^2)\,\Phi_1(w) = 0$$

$$\left(\varphi = \operatorname{arc\,tg}\frac{k_2 + k_1}{\nu(k_2 - k_1)}, \quad \beta = \frac{1}{2\pi}\ln\frac{k_1}{k_2}\right) \qquad (2.1)$$

Particular solutions of this equation in the neighborhood of the singular point $w = 0$ are given by

$$U_0(w) = F({}^1/_2 - i\beta_1,\ {}^1/_2 + i\beta_1,\ 1 + \varphi/\pi,\ w) \qquad (2.2)$$

$$V_0(w) = w^{-\varphi/\pi}\quad F({}^1/_2 - \varphi/\pi - i\beta_1,\ {}^1/_2 - \varphi/\pi + i\beta_1,\ 1 - \varphi/\pi,\ w)$$

In the neighborhood of the singular point $w = 1$, particular solutions of (2.1) are given by

$$U_1(w) = F({}^1/_2 - i\beta_1,\ {}^1/_2 + i\beta_1,\ 1 - \varphi/\pi,\ 1 - w)$$

$$V_1(w) = (1-w)^{\varphi/\pi} F({}^1/_2 + \varphi/\pi - i\beta_1, {}^1/_2 + \varphi/\pi + i\beta_1, 1 + \varphi/\pi,\ 1 - w) \qquad (2.3)$$

In the neighborhood of the point at infinity, the solutions of (2.1) take the form

$$U_\infty(w) = \left(\frac{1}{w}\right)^{1/2 - i\beta_1} F\left(\frac{1}{2} - i\beta_1,\ \frac{1}{2} - \frac{\varphi}{\pi} - i\beta_1,\ 1 - 2i\beta_1,\ \frac{1}{w}\right) \qquad (2.4)$$

$$V_\infty(w) = \left(\frac{1}{w}\right)^{1/2 + i\beta_1} F\left(\frac{1}{2} + i\beta_1,\ \frac{1}{2} - \frac{\varphi}{\pi} + i\beta_1,\ 1 + 2i\beta_1,\ \frac{1}{w}\right) \qquad w = \frac{c - b}{c + a}\,\frac{z + a}{z - b}$$

Utilizing relations (1.7) and (1.9), let us construct two linearly independent particular solutions of the homogeneous problem above for the functions $\Phi_1(w)$ and $\overline{\Phi}_1(w)$ in the region of slipping

$$\Phi_1^{+1}(w) = -\frac{i\,[k_2(\nu + i) - k_1(\nu - i)]}{2(k_1 + k_2)}\left(U_0(w) - \frac{\nu - i}{\nu + i}V_0(w)\right) \qquad (2.5)$$

$$\overline{\Phi}_1^{+1}(w) = -\frac{i\,[k_2(\nu + i) - k_1(\nu - i)]}{2(k_1 + k_2)}\,\bigl(U_0(w) - V_0(w)\bigr)$$

$$\Phi_1^{+2}(w) = -\frac{i\,[k_2(\nu+i)-k_1(\nu-i)]}{2(k_1+k_2)}(U_0(w)-V_0(w))$$
$$\overline{\Phi}_1^{+2}(w) = -\frac{i\,[k_2(\nu+i)-k_1(\nu-i)]}{2(k_1+k_2)}\left(U_0(w)-\frac{\nu+i}{\nu-i}V_0(w)\right) \qquad (2.6)$$

By the method of undetermined coefficients, we can obtain the particular solutions to the nonhomogeneous problem (1.7) to (1.9) for the functions $\Phi_1(w)$ and $\overline{\Phi}_1(w)$. These solutions take the form

$$\Phi_1^3(z) = \frac{iz}{2R(k_1+k_2)}, \qquad \overline{\Phi}_1^3(z) = \frac{-iz}{2R(k_1+k_2)} \qquad (2.7)$$

In accordance with the original formulation of the problem, the solution to (1.7) to (1.9) must be such that it vanishes for $z=\infty$, and for large $|z|$ the solution must be of the form

$$\Phi_1(z) = -\frac{X+iY}{2\pi z} + o\left(\frac{1}{z^2}\right), \qquad \overline{\Phi}_1(z) = -\frac{X-iY}{2\pi z} + o\left(\frac{1}{z^2}\right) \qquad (2.8)$$

where (Y, X) is the resultant vector of the external loading.

It is known that if $\Phi_1(z)$ is a solution, then $R(z)\,\Phi_1(z)$ is also a solution to the problem. To obtain a solution satisfying the previously listed conditions, we seek this solution in the form

$$\Phi_1(z) = P_1(z)\,\Phi_1^1(z) + Q_1(z)\,\Phi_1^2(z) + \Phi_1^3(z)$$
$$\overline{\Phi}_1(z) = P_1(z)\,\overline{\Phi}_1^1(z) + Q_1(z)\,\overline{\Phi}_1^2(z) + \overline{\Phi}_1^3(z) \qquad (2.9)$$

In (2.9), $P_1(z)$ and $Q_1(z)$ are first degree polynomials with undetermined coefficients. Upon determination of the coefficients for $P_1(z)$ and $Q_1(z)$, the relations in (2.9) take the form (2.10)

$$\Phi_1^+(x) = -\frac{1}{2R(k_1+k_2)}\left[\frac{D}{|D|}(\nu-i)\left(\frac{V_0'}{V_0}-x\right)\frac{V_0(x)}{V_0} - \nu\left(\frac{U_0'}{U_0}-x\right)\frac{U_0(x)}{U_0}\right] + \frac{ix}{2R(k_1+k_2)}$$

$$\overline{\Phi}_1^+(x) = -\frac{1}{2R(k_1+k_2)}\left[\frac{D}{|D|}(\nu+i)\left(\frac{V_0'}{V_0}-x\right)\frac{V_0(x)}{V_0} - \nu\left(\frac{U_0'}{U_0}-x\right)\frac{U_0(x)}{U_0}\right] - \frac{ix}{2R(k_1+k_2)}$$

$$\Phi_1^-(x) = -\frac{1}{2R(k_1+k_2)}\left[\frac{\overline{D}}{|D|}(\nu-i)\left(\frac{V_0'}{V_0}-x\right)\frac{V_0(x)}{V_0} - \nu\left(\frac{U_0'}{U_0}-x\right)\frac{U_0(x)}{U_0}\right] + \frac{ix}{2R(k_1+k_2)}$$

$$\overline{\Phi}_1^-(x) = -\frac{1}{2R(k_1+k_2)}\left[\frac{\overline{D}}{|D|}(\nu+i)\left(\frac{V_0'}{V_0}-x\right)\frac{V_0(x)}{V_0} - \nu\left(\frac{U_0'}{U_0}-x\right)\frac{U_0(x)}{U_0}\right] - \frac{ix}{2R(k_1+k_2)}$$

$$\left(D = k_2(\nu+i)-k_1(\nu-i), \quad |D| = \sqrt{\nu^2(k_2-k_1)^2+(k_1+k_2)^2}\right)$$

From these transformations we obtain two equations for the determination of the singular points a, c and b. These equations take the form (2.11)

$$\frac{\pi}{R(k_1+k_2)}\left(\frac{V_0'^2}{V_0^2}-\frac{V''}{2V_0}\right) = -Y, \qquad \frac{\pi\nu}{R(k_1+k_2)}\left[\left(\frac{U_0'^2}{U_0^2}-\frac{U_0''}{2U_0}\right)-\left(\frac{V_0'^2}{V_0^2}-\frac{V''_0}{2V_0}\right)\right] = -X$$

where U_0, U_0', U_0'', V_0, V_0', V_0'' are coefficients in the expansion of the functions $U_0(z)$ and $V_0(z)$ for large $|z|$.

To obtain the functions $\Phi_1(z)$ and $\bar{\Phi}_1(z)$ in the region of sticking, we make use of the analytic continuation of $U_0(z)$ and $V_0(z)$ beyond the region of convergence of the series. The formulas for the analytic continuation of $U_0(z)$ and $V_0(z)$ are given by [9]

$$U_0(z) = \beta_{11} e^{\pm\pi i\alpha} U_\infty(z) + \beta_{12} e^{\pm\pi i\beta} V_\infty(z)$$

$$V_0(z) = \beta_{21} e^{\mp\pi i(\gamma-\alpha-1)} U_\infty(z) + \beta_{22} e^{\mp\pi i(\gamma-\beta-1)} V_\infty(z) \tag{2.12}$$

$$(\alpha = 1/2 - i\beta_1,\ \beta = 1/2 + i\beta_1,\ \gamma = 1 + \varphi/\pi)$$

$$\beta_{11} = \frac{\Gamma(1+\varphi/\pi)\,\Gamma(1+2i\beta_1)}{2i\beta_1\Gamma(1/2+i\beta_1)\,\Gamma(1/2+\varphi/\pi+i\beta_1)}, \qquad \beta_{12} = \bar{\beta}_{11}$$

$$\beta_{21} = \frac{(1/2-\varphi/\pi+i\beta_1)\,\Gamma(1-\varphi/\pi)\,\Gamma(1+2i\beta_1)}{2i\beta_1\Gamma(1/2+i\beta_1)\,\Gamma(3/2-\varphi/\pi+i\beta_1)}, \qquad \beta_{22} = \bar{\beta}_{21} \tag{2.13}$$

Thus, we can write the solution to the problem over the entire contact region. The loading components for the contact region are given by:

1) in the slipping region

$$P(x) = -\frac{1}{R|D|}\left(\frac{V_0'}{V_0} - x\right)\frac{V_0(x)}{V_0}, \qquad T(x) = -\frac{\nu}{R|D|}\left(\frac{V_0'}{V_0} - x\right)\frac{V_0(x)}{V_0} \tag{2.14}$$

2) in the sticking region

$$P(x) = -\frac{1}{2R\sqrt{k_1k_2}}\left(\frac{V_0'}{V_0^2} - \frac{x}{V_0}\right)(\beta_{21}U_\infty(x) + \beta_{22}V_\infty(x))$$

$$T(x) = -\frac{\nu}{2R\sqrt{k_1k_2}}\left[\left(\frac{V_0'}{V_0^2} - \frac{x}{V_0}\right)(\beta_{21}U_\infty(x) + \beta_{22}V_\infty(x)) - \right.$$
$$\left. - \left(\frac{U_0'}{U_0^2} - \frac{x}{U_0}\right)(\beta_{11}U_\infty(x) + \beta_{12}V_\infty(x))\right] \tag{2.15}$$

The third equation for the determination of the ends of the contact region and of the division point between the two contact zones is obtained from the requirement that the stresses at $z = -a$ be bounded.

3. Let us consider the case of particular values of the elastic constants. For a perticular set of elastic constants, we have $k_1 = k_2 = k$, $\beta_1 = 0$ and $\varphi = \frac{1}{2}\pi$. We seek a solution having an integrable singularity at $z = -a$ and bounded at $z = c$ and b. In that case, the parameters in Gauss' equation will be $\alpha = \frac{1}{2}$, $\beta = \frac{1}{2}$, and $\gamma = \frac{3}{2}$ while Gauss' equation takes the form

$$w(1-w)\,\Phi_1''(w^{-1/2}) + (3/2 - 2w^{-1/2})\,\Phi_1'(w^{-1/2}) - 1/4\,\Phi_1(w) = 0 \tag{3.1}$$

Solutions which are analytic in the neighborhoods of the singular points $w = 1$, 0 and ∞ are given by

$$U_0(w) = F(1/2, 1/2, 3/2, w) = w^{-1/2}\arcsin(w)^{1/2}$$
$$V_0(w) = w^{-1/2}F(0, 0, 1/2, w) = w^{-1/2}$$
$$U_1(w) = F(1/2, 1/2, 1/2, 1-w) = w^{-1/2} \tag{3.2}$$
$$V_1(w) = (1-w)^{1/2}F(1, 1, 3/2, 1-w) = w^{-1/2}\arccos(w)^{1/2}$$
$$U_\infty(w) = V_\infty(w) = w^{-1/2}F\left(1/2, 0, 1, \frac{1}{w}\right) = w^{-1/2}$$

The solution of our problem will be sought in the same form as in the general case. In the slipping region, the limiting forms of the functions $\Phi_1(z)$ and $\overline{\Phi}_1(z)$ are given by

$$\Phi_1^{\pm}(x) = \frac{1}{4Rk}\left[\pm i\,(\nu - i)\left(\frac{V_0'}{V_0} - x\right)\frac{V_0(x)}{V_0} + \nu\left(\frac{U_0'}{U_0} - x\right)\frac{U_0(x)}{U_0} + ix\right]$$
$$\overline{\Phi}_1^{\pm}(x) = \frac{1}{4Rk}\left[\pm i\,(\nu + i)\left(\frac{V_0'}{V_0} - x\right)\frac{V_0(x)}{V_0} + \nu\left(\frac{U_0'}{U_0} - x\right)\frac{U_0(x)}{U_0} - ix\right] \qquad (3.3)$$

Let us now write the expressions for the desired functions in the sticking region, taking into account the change occurring in the functions $U_0(w)$ and $V_0(w)$ upon passing the point $w = 1$. The function $V_0(w)$ is defined everywhere so that there is no need for any continuation. The function $U_0(w)$ takes the following form on the two sides of the cut along the real axis from 1 to ∞:

$$U_0^{\pm}(w) = w^{-1/2}\arccos\left(e^{\mp 1/2\pi i}(w-1)^{1/2}\right) \qquad (3.4)$$

We can now write the expressions for the desired functions in the sticking region

$$\Phi_1^{\pm}(x) = \frac{1}{4Rk}\left[\pm i\,(\nu - i)\left(\frac{V_0'}{V_0^2} - \frac{x}{V_0}\right)w^{-1/2} + \right.$$
$$\left. + \nu\left(\frac{U'_0}{U_0^2} - \frac{x}{U_0}\right)\arccos\left(e^{\mp 1/2\pi i}(w-1)^{1/2}\right)w^{-1/2} + ix\right]$$
$$\overline{\Phi}_1^{+}(x) = \frac{1}{4Rk}\left[\pm i\,(\nu + i)\left(\frac{V_0'}{V_0^2} - \frac{x}{V_0}\right)w^{-1/2} + \right. \qquad (3.5)$$
$$\left. + \nu\left(\frac{U_0'}{U_0^2} - \frac{x}{U_0}\right)\arccos\left(e^{\mp 1/2\pi i}(w-1)^{1/2}\right)w^{-1/2} - ix\right]$$

The formulas for the load components are:

1) in the slipping region

$$P(x) = \frac{\Omega(x)}{2Rk}\left(\frac{V_0'}{V_0^2} - \frac{x}{V_0}\right)$$
$$T(x) = \frac{\nu\Omega(x)}{2Rk}\left(\frac{V_0'}{V_0^2} - \frac{x}{V_0}\right) \qquad \Omega(x) = \left(\frac{c-b}{c+a}\,\frac{x+a}{x-b}\right)^{-1/2} \qquad (3.6)$$

2) in the sticking region

$$P(x) = \frac{\Omega(x)}{2Rk}\left(\frac{V_0'}{V_0^2} - \frac{x}{V_0}\right),$$
$$T(x) = \frac{\nu\Omega(x)}{2Rk}\left[\left(\frac{V_0'}{V_0^2} - \frac{x}{V_0}\right) - \frac{1}{2}\left(\frac{U_0'}{U_0^2} - \frac{x}{U_0}\right)\ln\frac{\sqrt{w} - \sqrt{w-1}}{\sqrt{w} - \sqrt{w-1}}\right] \qquad (3.7)$$

Here U_0, U_0', V_0 and V_0' are coefficients in the expansion of the functions $U_0(z)$ and $V_0(z)$ for large $|z|$. Eqs. (2.11) for the determination of the end points and the division point in the contact region become

$$\frac{\pi(a+b)(3b-a)}{16Rk} = -Y \qquad (3.8)$$
$$\frac{\pi}{4Rk}\left\{(a+b)^{1/2}(b-c)^{1/2}(c+a+4b)\left[\operatorname{Arsh}\left(\frac{b-c}{c+a}\right)^{-1/2}\right]^{-1} - \right.$$
$$\left. - 2(a+b)(b-c)\left[\operatorname{Arsh}\left(\frac{b-c}{c+a}\right)^{-1/2}\right]^{-2}\right\} = X$$

Upon determining V_0 and V_0' in the formula for the normal load we obtain

$$P(x) = \frac{1}{2Rk}\left(\frac{b-x}{x+a}\right)^{1/2}\left(\frac{a+b}{2} + x\right) \tag{3.9}$$

In order that the solution be bounded at the point $z = -a$, set $a = b$. Then (3.9) becomes

$$P(x) = \frac{1}{2Rk}\sqrt{a^2 - x^2} \tag{3.10}$$

The formula for determining the ends of the contact region takes the form

$$a = \sqrt{RP\,\frac{x+1}{\mu\pi}} \tag{3.11}$$

The last two formulas coincide with the results of Glagolev [6].

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